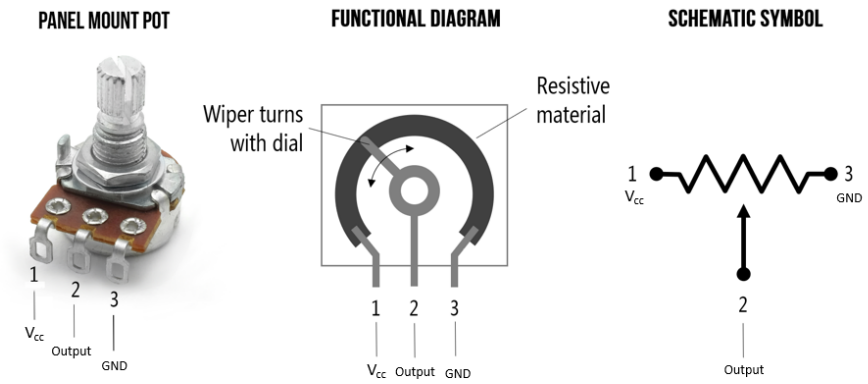
Exercise set 11 – Sensors – Solutions

Exercise 1 (Already given in the lecture)

Consider a potentiometer covering the range of $350°$ powered with $\pm 10\text{V}$. Considering an electrical noise of 20mV:

1. Give the maximum resolution that can be obtained.
2. Suggest an A/D (analog to digital) converter for this potentiometer. Give the:
   - number of divisions
   - number of bits
   - resolution

Reminder: A potentiometer is a 3 terminal variable resistor in which the resistance is varied.



Exercise 1 - Solution

1. The maximum resolution that can be obtained is:

$$\frac{20\ mV}{20\ V} 350° = 0.35°$$

2. The number of divisions that the A/D converter must have is:

$$\frac{20\ V}{20\ mV} = 1000\ div$$

   A converter with a resolution of 1000 divisions must be at least 10 bits, i.e. 1024 divisions ($2^9 = 512 < 1000$ but $2^{10} = 1024 > 1000$). The resolution of this converter is:

$$\frac{20\ V}{1024\ div} = 19.53\ mV/div$$

Exercise 2

Consider a force sensor with 100N range, outputting an electrical signal between 0 and 100mV for the full range. The bandwidth of this force sensor is 100Hz. The signal is conditioned and adapted to the analog input of an acquisition card by an amplifier (amplification and filtering). The A/D converter of this acquisition card has a resolution of 10bit over an input range of 0 to 5V.

1. Determine the force resolution using (in terms of N/division) the force sensor directly on the analog input without passing the signal through an amplifier.
2. Suggest an amplification to use the analog input to its full range of 5V.
3. Determine the new force resolution.

4. Propose a sampling period to acquire the force signal without deteriorating it on the frequency level in terms of theoretical limits and the practical aspects.
5. Explain the acquisition constraints on the force signal using this acquisition card on a PC with a sampling frequency of $50\mathrm{Hz}$.

Exercise 2 - Solution

1. The force resolution using no additional electronics is:

$$\frac{100\ N}{100\ mV}\frac{5\ V}{1024\ div} = \ 4.88\ N/div$$

This means that the smallest measurable force (excluding the effect of noise) is $4.88\mathrm{N}$.

2. A conditioning amplifier must be used to upgrade the full range of the force sensor to the full range of the analog input of the acquisition card. The gain of this amplifier must make it possible to go from $100\mathrm{mV}$ to $5\mathrm{V}$. This therefore corresponds to a gain of 50.

3. The new resolution of the force measurement is:

$$\frac{100\ N}{5\ V}\frac{5\ V}{1024\ div} = 97.6\ mN/div$$

which is 50 times better because we use the full range of the force sensor on $10\mathrm{bit}$. (This of course excluding the effect of noise.)

4. According to Shannon's theorem, we must take a sampling frequency f_s at least twice as high as the passband, i.e. $f_s > f_{\lim} = 200\mathrm{Hz}$. $f_{\lim}$ is a bare minimum.

   It should be noted that the amplifier must also make it possible to filter the noise which goes beyond the bandwidth of the force sensor. Therefore, In practice, we want a sampling frequency 5 to 10 times higher than the pass band, i.e. $500\mathrm{Hz} < f_s < 1000\mathrm{Hz}$.

   We could reasonably choose a low pass filter with a cutoff frequency between $500\mathrm{Hz}$ and $1\mathrm{kHz}$. This gives a sampling period T_s between $1\mathrm{ms}$ and $2\mathrm{ms}$, with a limiting sampling period of $T_{\lim} = 5\ \mathrm{ms}$.

5. By using an acquisition on a PC limited to $50\mathrm{Hz}$ (i.e. a sampling period of $20\mathrm{ms}$):
   - The force signal must not contain frequency components greater than $25\mathrm{Hz}$ if we want to respect Shannon's theorem.
   - The signal must not contain frequency components greater than $5\mathrm{Hz}$ if we want to respect the empirical value of factor 10 and to be able to reproduce faithfully the measured signal.

Exercise 3

Consider a rotary incremental encoder to measure the position of a rotary arm. This encoder has 1000 lines (physical holes) per revolution. A counter for quadrature-phase signals is used as an interface to this encoder.

1. Give the resolution in position of the robot arm in the absence of reducer.
2. Give the resolution in position of the robot arm in the presence of a reducer with a reduction ratio of 30.
3. Give the number of tracks that an equivalent optical absolute encoder on the load side should have to give the same resolution.
4. Let's say that we use an absolute optical encoder with a $16\mathrm{bit}$ counter. What would be the maximum measured position range?

Exercise 3 – Solution

1. The position resolution in direct actuation (i.e. without a reducer) is:

$$\frac{360°}{1000 \cdot 4} = 0.09°/div$$

The factor 4 results from the phase quadrature.

2. In the presence of a reducer of ratio 30, the position resolution becomes:

$$\frac{360°}{1000 \cdot 4 \cdot 30} = 0.003°/div$$

which is 30 times better than without the reducer.

3. A resolution of $0.003°/\text{div}$ is equivalent to dividing the full position range (360°) into:

$$\frac{360°}{0.003°/div} = 120\ 000\ div$$

We know that:

- 16bit corresponds to 2^{16} = 65 536 div < 120 000div
- 17bit corresponds to 2^{17} = 131 072 div > 120 000div

The equivalent absolute sensor should have at least 17 tracks, which gives an idea of the cost for its manufacturing.

4. 16bit corresponds to 2^{16} = 65 536div, so the counter can encode 65 536 positions, from 0 to 65535. The maximum position value is therefore:

$$\theta_{max} = 65535 \cdot 0.003° = 196.305°$$

This counter is not sufficient to measure the 360° of the robot arm; it will either be necessary to change the counter (and therefore probably the acquisition card), or to manage the size overruns of the counter in a software way.

Exercise 4

We want to realize the angular axis of a robot with a rotary motor and a reducer. An absolute sensor of 20bit is used over a range of 360°. Initially, the actuation is direct.

1. Give the linear resolution at the extremity of an arm of 350mm.

We now add a reducer with a reduction ratio of 3.

2. Give the linear resolution at the extremity of an arm of 350mm.
3. Is the absolute sensing still valid in case we use this reducer?

Exercise 4 – Solution

1. The angular position resolution in direct actuation (i.e. without the use of a reducer) is:

$$\frac{360°}{2^{20}} = 3.43 \times 10^{-4}\ °/div$$

The linear resolution at the end of an arm of 350mm is:

$$\frac{2\pi \cdot 350mm}{2^{20}} = 2.1\mu m/div$$

2. The position resolution with a reducer of ratio 3 is 3 times smaller:

$$\frac{360°}{2^{20} \cdot 3} = 1.14 \times 10^{-4}\ °/div \text{ (angular position resolution)}$$

$$\frac{2\pi \cdot 350mm}{2^{20} \cdot 3} = 0.7\ \mu m/div \text{ (linear position resolution)}$$

3. With a reducer of ratio 3, the sensor is no longer absolute beyond $\frac{360°}{3} = 120°$